Optimization in geotechnics

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1 Optimization including Uncertainty, Sensitivity and Robustness Analysis

Today the rock mechanical engineer is forced to develop safe and economic designs for systems, which are characterized by many influencing factors (technical parameters, rock mass parameters, economical parameters, environmental parameters, geometrical parameters etc.). In such systems (see Fig. 1) it is becoming more and more difficult to find optimal solutions and to identify those parameters, which are really decisive. Therefore, classical procedures like parameter studies, 'trial-and-error'-procedures and simple parameter fitting is replaced more and more by so-called mathematical optimization techniques (Konietzky & Schlegel 2013).

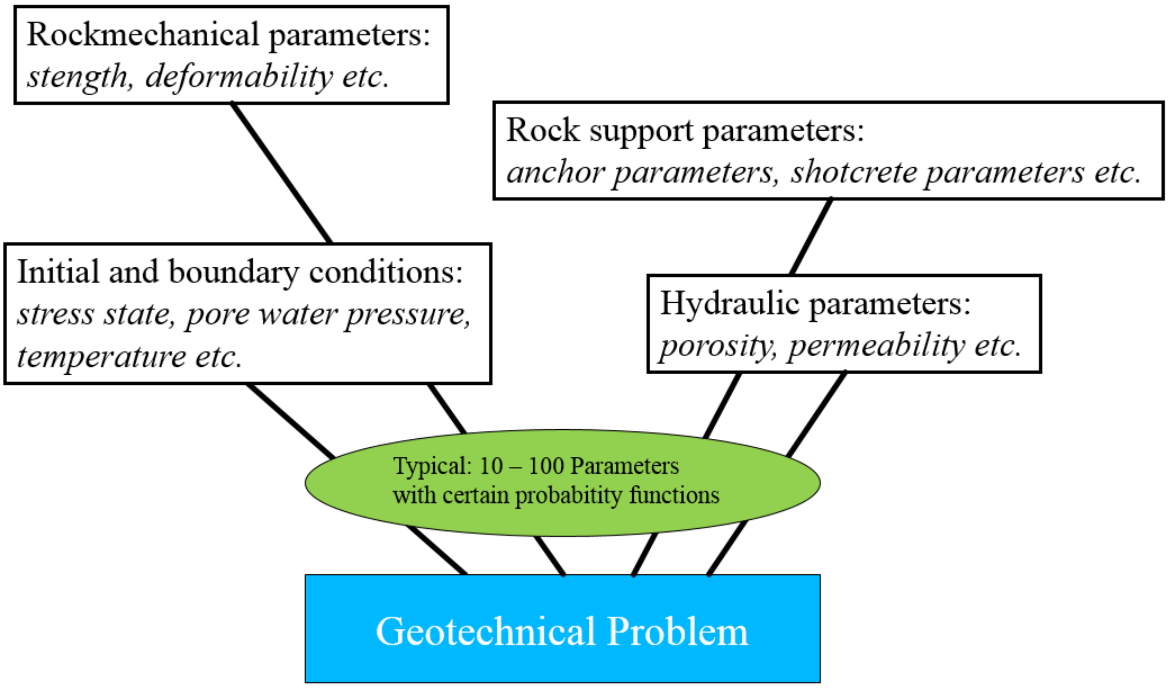

Fig. 1: Typical geotechnical parameter situation in respect to optimization and related procedures

These techniques involve the following analysis methods:

- Sensitivity analysis:
  Within a sensitivity analysis the influence of individual input parameters on the model output is investigated. Based on the sensitivity analysis insignificant parameters can be excluded from further design and interaction of different parameters can be revealed. Enhanced sensitivity analysis includes the checking of the prediction quality of response variation and the deduction of MOP (**M**etamodel of **O**ptimal **P**rognosis).

- Uncertainty analysis:
  Uncertainty analysis investigates the uncertainty of input variables and their impact on the model responses. Uncertainty estimates are used to assess the confidence of model results.

- Robustness analysis:
  Within the robustness analysis it is estimated how robust the model response is in relation to uncertainty in the input parameters or in other words, how stable is our model response if the input variables scatter in a certain manner. Robustness evaluation is mainly performed for the best design, which may be obtained by previous optimization.

- Reliability analysis:
  Reliability analysis investigates the boundary violation of output due to the variability of input parameters. Reliability analysis can either predict the failure probability or proof that all constraints scatter within defined boundaries.

- Optimization:
  Optimization means the mathematical based selection of the best fitting objects in respect to defined criteria. This leads to the maximizing or minimizing of functions by choosing input values or functions from a certain parameter set or range. Different deterministic and stochastic optimization schemes are available. Popular deterministic approaches are response surface methods, hill climbing methods or gradient based strategies. Typical stochastic approaches are: evolutionary algorithms, neural network approaches, particle swarm algorithms or the Fuzzy logic theory based methods. Of great importance is multi-objective optimization, where an optimum is searched not only for one parameter or function, but for several and sometimes also contradictory objective functions (Pareto-Optimization).
  The optimization procedure can be optimized in terms of reducing solver calls by using the MOP as pre-search tool.

- Parameter identification:
  Parameter identification is a reverse optimization procedure (inverse parameter identification), where parameters and/or constitutive relations are determined in such a way, that the model with this parameters/constitutive relations predicts results, which are close to measurements or observations (automatic model calibration).

- Data mining:
  Data mining is a process to extract hidden information from large amounts of existing data. Therefore mainly statistical algorithms are used. Data mining is currently a very important issue in many research and application fields.

The application of the above mentioned mathematical based procedures demands the existence or development of a model (analytical, empirical or numerical), which describes the rock mechanical problem (e.g. a tunnel construction, a slope instability, the stress-strain behavior of a rock sample etc.). Such a model is characterized by input and output parameters or functions. Although the huge potential of optimization techniques is recognized and already extensively used in engineering in general, the breakthrough in geomechanics has not yet taken place. Only on the academic level several publications illustrate the potential, e.g. Konietzky & Schlegel 2013, Beiki et al. 2010, DeGagne 2011, Garvey & Ozbay 2011, Keshavarzi & Jahanbakhski 2011, Kim et al. 2012, Moel et al. 2012, Shao & Su 2010, Tran & Abousleiman 2010, Wainwright et al. 2012, Xu et al. 2013, Yoon 2007.

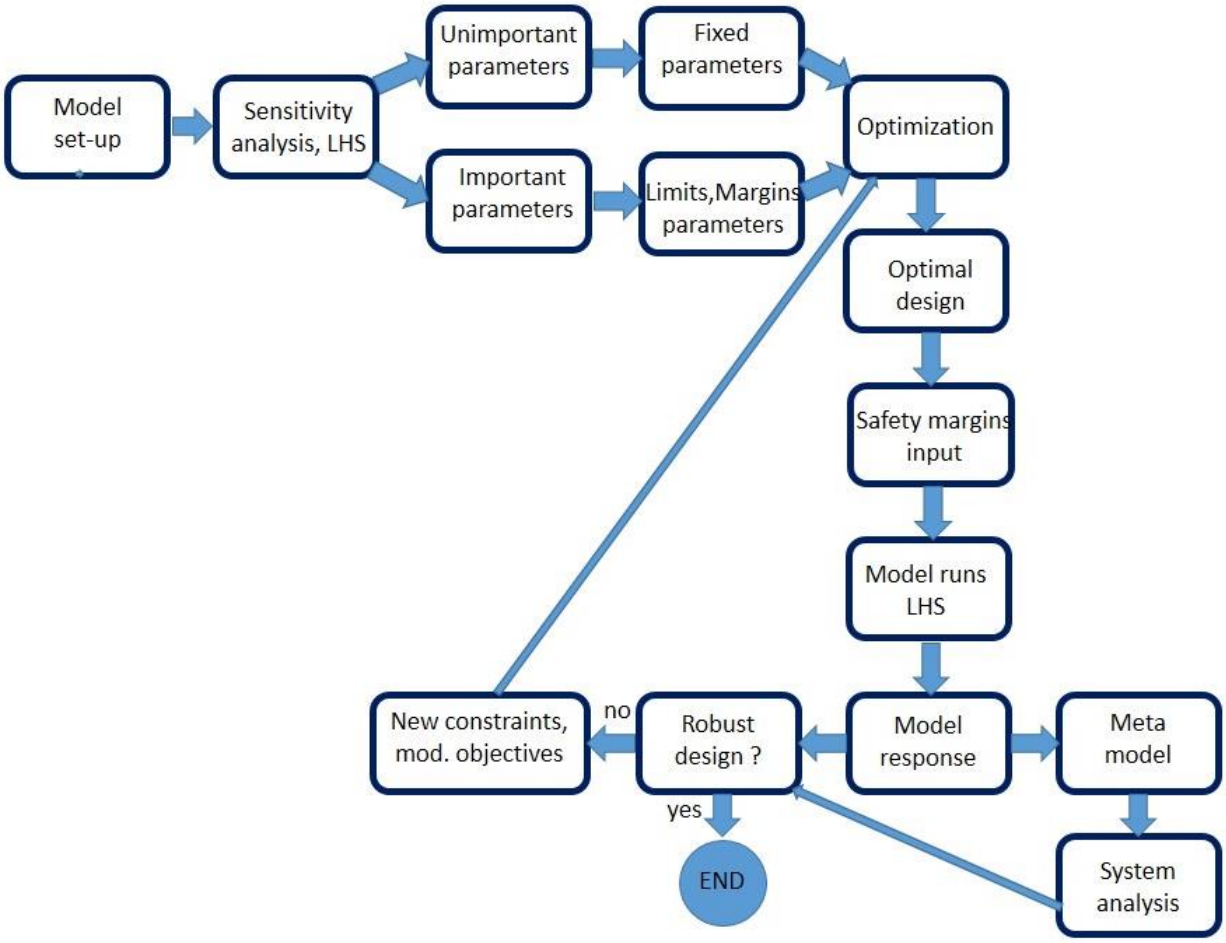

Fig. 2: Typical RDO-scheme

The basic strategy of solving rock mechanical problems by using optimization schemes consists of 2 steps: Sensitivity analysis first, followed by optimization.

Within the sensitivity analysis it is analyzed which input parameters are relevant for the considered output. On the basis of the evaluation of the sensitivity analysis unimportant parameters can be eliminated during the subsequent optimization, which makes the optimization easier to perform, faster and more robust. The rock mechanical problems under consideration often contain much more than ten input-parameters. Also, their influence can be non-linear and the input parameters can influence each other. This all makes it impossible to estimate the influence of the input parameters on the output by just the engineering judgment due to experience or simple parameter studies. The most logical alternative is the use of a mathematical based sensitivity study. This has to be based on stochastic sample algorithms, which means that sets of input parameters have to be created, which should cover the whole possible range of parameter constellations. The most straightforward procedure is the Monte Carlo method, but it leads to an extremely high number of model sets. Therefore, alternatives, like the Latin Hypercube Sampling, were developed, which require fewer sampling points to cover the whole design space. Our experience has shown, that for even high order design spaces (several dozen of input parameters) only about 50 up to a few hundred model runs are necessary, if Latin Hypercube Sampling is used.

A comprehensive mathematical based optimization procedure is the so-called **R**obust **D**esign **O**ptimization (RDO), which contains several steps and loops as illustrated in Fig. 2. Besides a robust optimized design so-called meta-models can be created to perform system analysis. Reliability analysis or stochastic stability analysis (failure probability) is often specified in terms of so-called sigma-levels (σ-level). This measure is based on the normal distribution with corresponding mean values and standard deviations. Table 1 and Figure 3 illustrate the σ-levels.

Tab. 1. Sigma-levels and their corresponding values.

σ-level	Intact elements (%)	Failed elements (%)	Failure frequency
1	68	32	1 in 3
2	95	5	1 in 22
3	99.73	0.27	1 in 370
4	99.9937	0.0063	1 in 15 787
5	99.999943	0.000057	1 in 1 744 160
6	99.9999998	0.0000002	1 in 506 797 346

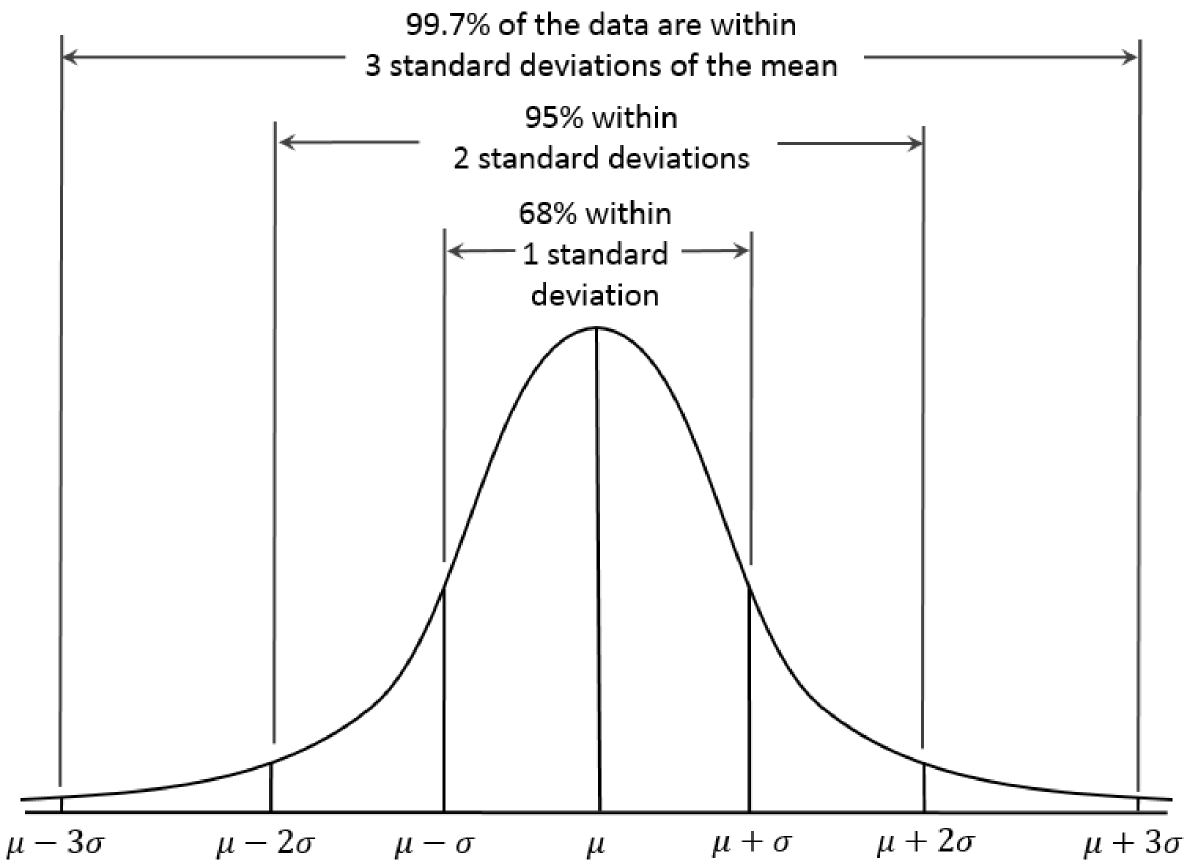

Fig. 3: Illustration of Sigma-levels based on the Gaussian normal distribution

Below a simplified rock mechanical problem is used to illustrate some steps inside a RDO. In the shown case, the whole process can be subdivided into five steps:

Step 1: Conversion of the rock mechanical problem into a numerical model

The rock mechanical problem may be the design of an anchor scheme for the roof of an underground chamber with rectangular cross section. Corresponding initial and boundary conditions, constitutive laws for the rock mass, the anchors and the interaction between anchors and rock mass have to be specified. To demonstrate the whole procedure a simple 2-dimensional numerical model as shown in Fig. 4 is used.

The model represents half of a chamber 10 m wide and 5 m high. Due to the symmetry conditions, a half-space model was used. The model has a vertical symmetry line at the left boundary and contains 5 roof anchors. The rock mass was modeled using the classical elasto-plastic Mohr-Coulomb model with tension cut-off and non-associated flow rule. The virgin principal stress state is characterized by 10 MPa vertical stress and 5 MPa horizontal stress. The outer model boundaries are fixed in the normal directions.

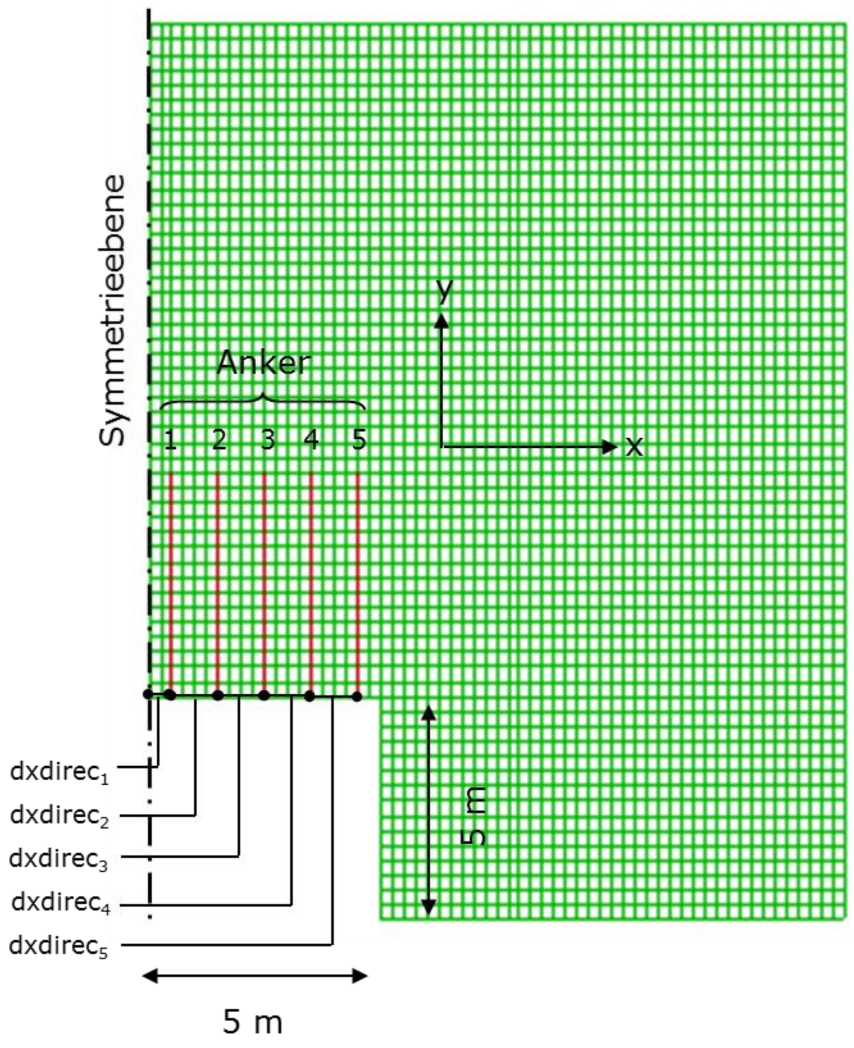

Fig. 4: Numerical model deduced from the rockmechanical problem of roof anchorage (mesh with 5 roof anchors) and variable anchor location.

Step 2: Specification of input and output parameters

According to the specified constitutive laws a basic parameter-set and a corresponding parameter range for the sensitivity analysis have to be defined (Tables 2 and 3)

Tab. 2: Basic parameter set for rock mass, anchor and grout.

Parameter	Description	Unit	Value
	Rock Mass		
E	Young's-modulus	Pa	10^9
ν	Poisson's ratio		0.3
φ	Friction angle	°	30
ψ	Dilation angle	°	10
c	Cohesion	Pa	10^5
σ_t	Uni. tens. strength	Pa	0
ρ	Density	kg/m³	2500
	Anchor		
length_anker	Anchor length 1 – 5	m	5
dxdirec	Distance between anchors	m	1
e_anker	Young's modulus of anchor steel	Pa	10^{10}
yield_anker	Yield point of anchor steel	Pa	10^7
radius_anker	Anchor radius	m	0.01
	Interaction between anchors and rock mass (FLAC, 2006)		
cs_ncoh / cs_scoh	Cohesion normal / shear	Pa	10^5 / 10^5
cs_sfric	Friction angle	°	10
cs_nstiff / cs_sstiff	Grout stiffness normal / shear	Pa	10^7 / 10^7
grout_thick	Grout thickness	M	0.01

Tab. 3: Input parameters with lower and upper bounds for sensitivity analysis.

Name	Description	Unit	Lower Bound	Upper Bound
length_anker	length of anchor	m	1.0	5.0
radius_anker	radius of anchor	m	0.01	0.1
e_anker	E-Modulus of anchor	Pa	10^{10}	$2.0*10^{11}$
cs_ncoh	grout normal cohesion	Pa	10^{5}	$5.0 *10^{6}$
cs_scoh	grout shear cohesion	Pa	10^{5}	$5.0 *10^{6}$
cs_sfric	friction angle	°	10	60
cs_sstiff	Grout shear stiffness	Pa	10^{7}	10^{8}
cs_nstiff	Grout normal stiffness	Pa	10^{7}	10^{8}
yield_anker	yield stress of anchor material	Pa	10^{8}	10^{9}
grout_thick	grout thickness	m	0.01	0.1

Step 3: Coupling between optimization tool and numerical code (master-slave-modus)

The numerical model and the optimization tool have to communicate with each other in a master-slave mode. For the given example the tools optislang [dynardo 2014] and FLAC [ITASCA 2014] were used. The optimization tool has to specify and change, respectively, the input-parameters for the individual model runs and has to get specific model response data from the numerical model for further evaluation (Fig. 5). This communication can be performed by special interfaces or simply by using ASCII-files, which will be written and read by the two codes for communication.

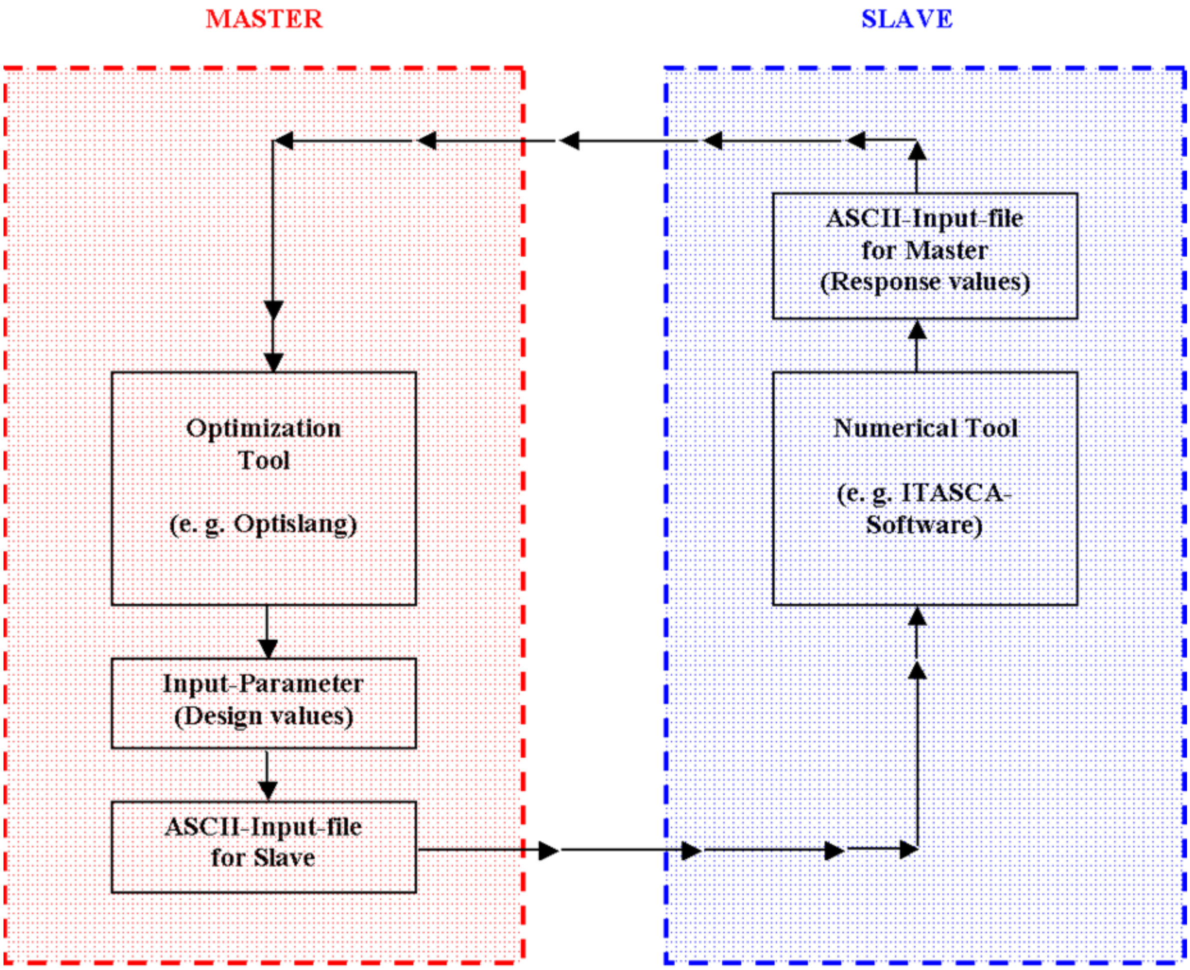

Fig. 5: Work flow of optimization tool and numerical solver in form of a master-slave-modus.

Step 4: Execution of sensitivity analysis

First of all, a sensitivity analysis was performed to investigate the impact of the individual parameters on the roof subsidence. The most important parameters according to Tab. 3 were used. A stochastic sampling method, the Latin Hypercube sampling, was used to define the input parameter sets. Only 50 model runs were necessary to derive the linear and quadratic correlation coefficients and coefficients of determinations, respectively.

The Anthill-Plot for the anchor length versus the obtained maximum roof settlement is shown in Figure 6. The example is based on the simplified assumption, that all anchors have the same length and that the distance between the anchors is fixed. The plot of the coefficient of determination of the maximum roof settlement (Fig. 7) shows, that 60% of the roof settlements can be explained by the anchor length alone. Similar indications are given be the correlation coefficient as shown in Fig. 8. All other input parameters together have less than 40% influence. As expected the maximum roof subsidence decrease with increasing anchor length.

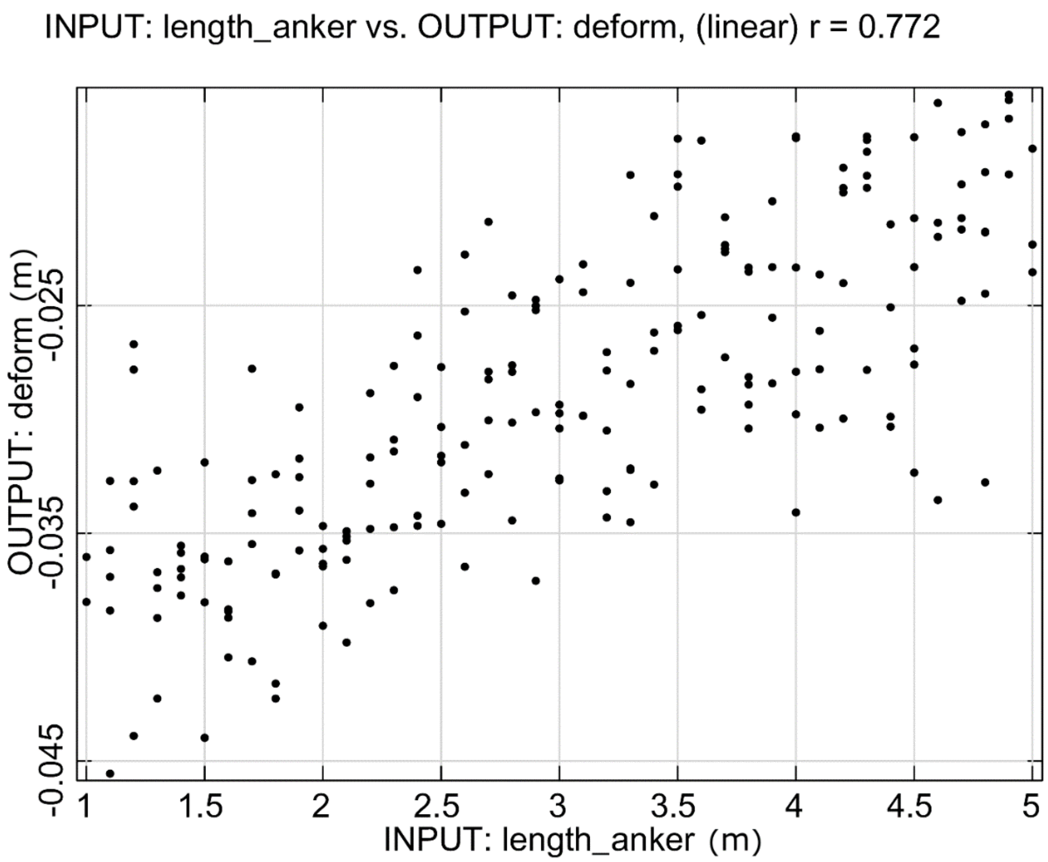

Fig. 6: Anthill-plot: anchor length versus output (maximum roof settlement).

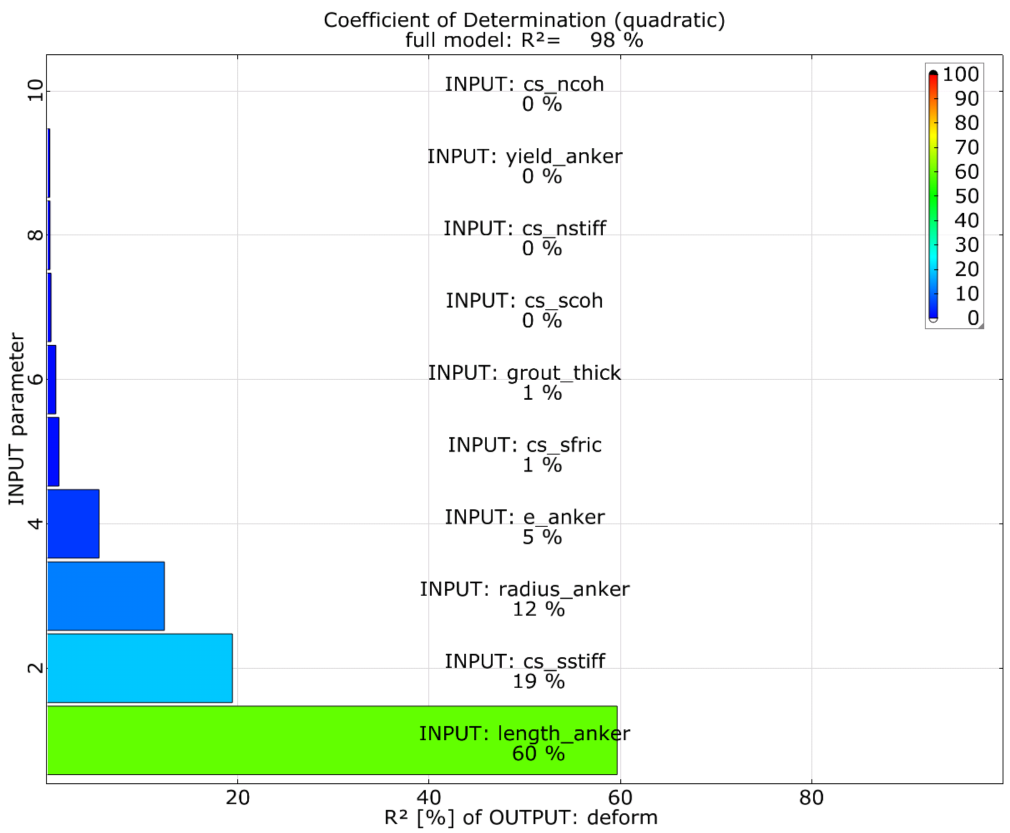

Fig. 7: Coefficient of determination

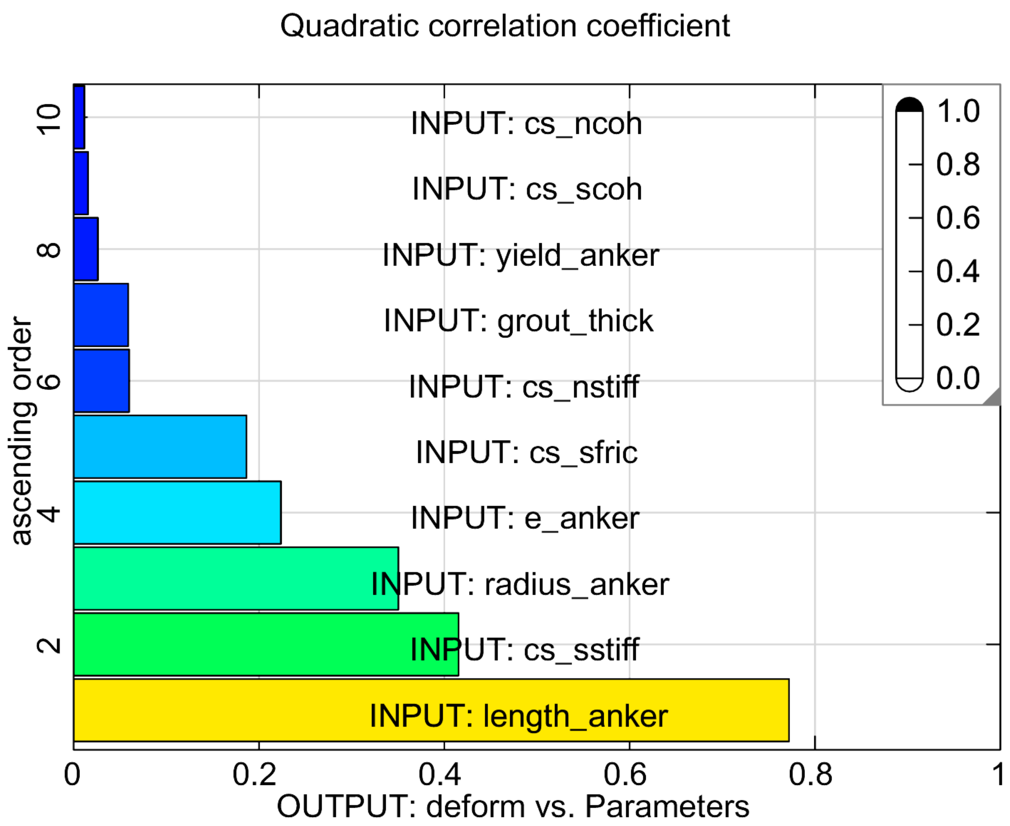

Fig. 8: Quadratic correlation coefficient.

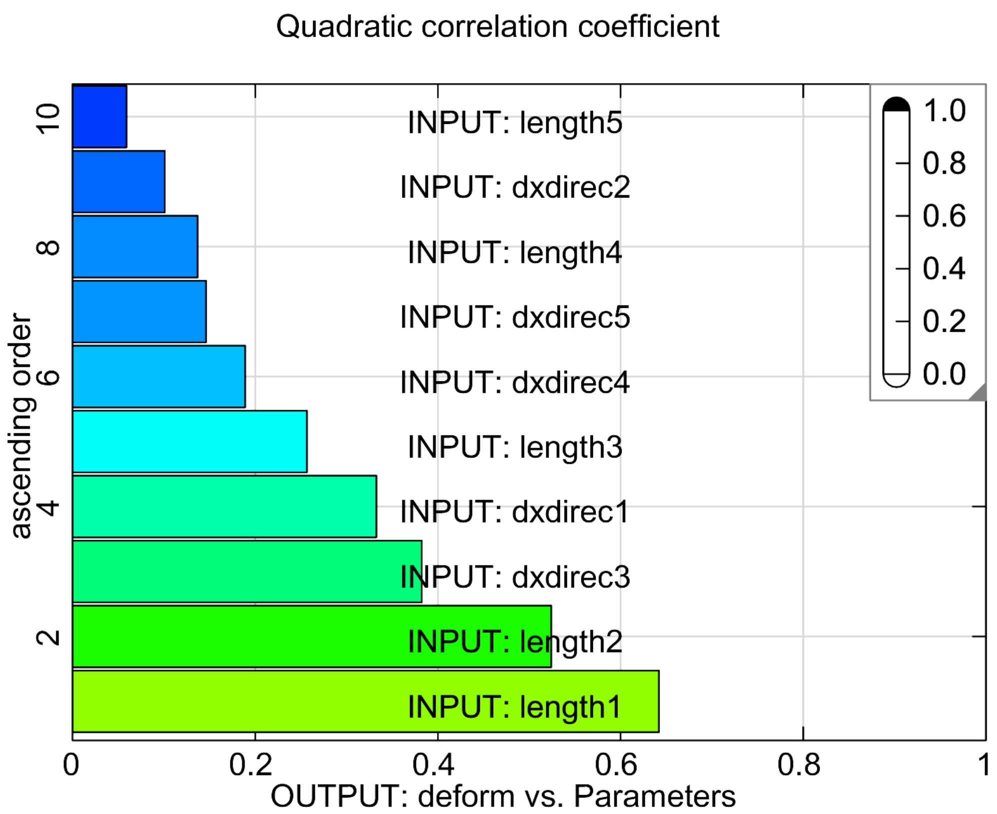

Fig. 9: Quadratic correlation coefficient.

Based on the knowledge, that the anchor length plays a dominant role, a second sensitivity analysis was performed, there both, the parameters for the rock mass as well as the parameters for the interaction between the rock mass and the anchors were fixed. Instead of fixed anchor locations, now the anchors are allowed to move along the roof and the anchor length can be different for each anchor within the values of 1 m to 5 m (Table 4). This sensitivity analysis shows that the anchors 1 and 2 have by far the strongest influence on the roof subsidence. The length of the anchor 5 (nearest to the side wall) shows only insignificant correlation to the maximum roof subsidence (Figures 9 and 10).

Tab. 4: Input parameters with lower and upper bounds for sensitivity analysis.

Name	Description	Unit	Lower Bound	Upper Bound
length1	length of anchor 1	m	1.0	5.0
length2	length of anchor 2	m	1.0	5.0
length3	length of anchor 3	m	1.0	5.0
length4	length of anchor 4	m	1.0	5.0
length5	length of anchor 5	m	1.0	5.0
dxdirec1	distance between symmetry axis and anchor 1	m	0.25	2.5
dxdirec2	distance between anchor 1 and 2	m	0.5	2.5
dxdirec3	distance between anchor 2 and 3	m	0.5	2.5
dxdirec4	distance between anchor 3 and 4	m	0.5	2.5
dxdirec5	distance between anchor 4 and 5	m	0.5	2.5

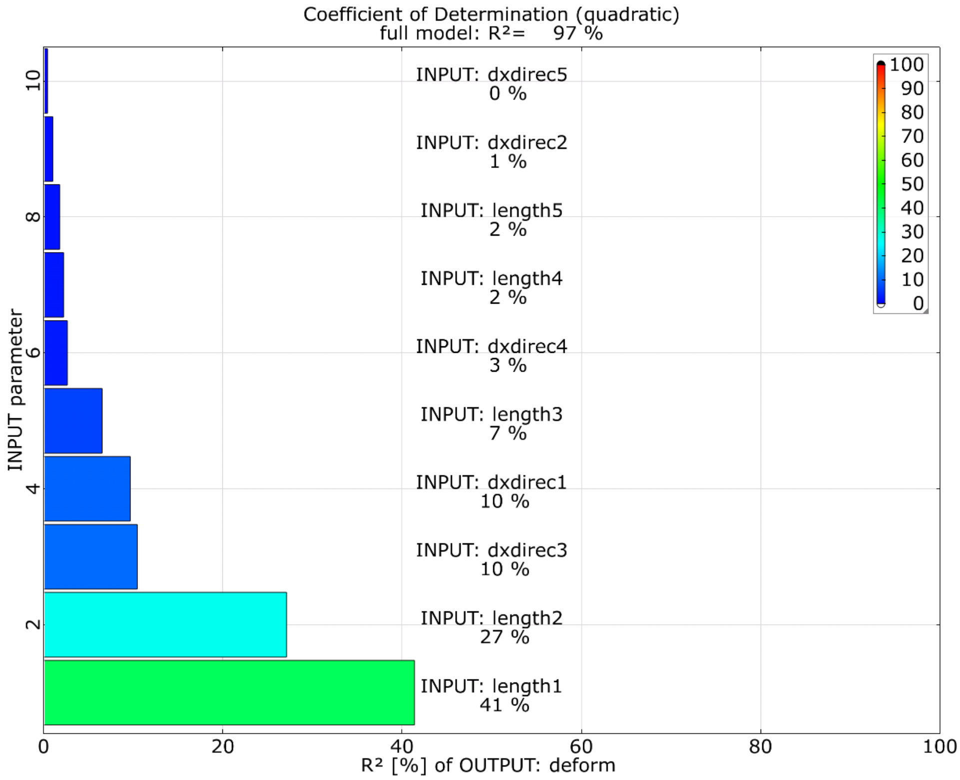

Fig. 10: Coefficients of determination.

Step 5: Pareto-Optimization

After getting detailed knowledge about the sensitivity of the input parameters, optimization can start. First of all an objective function has to be specified. Here, for simplicity two contradictory objectives were considered: besides the technical (safety) objective f1 (= maximum allowable roof subsidence) a second (economic) objective f2 (= minimum of total anchor length) was defined:

$$f_1 = \max(\text{roof subsidence}) \rightarrow \min,$$
$$f_2 = \sum \text{anchor length} \rightarrow \min,$$

with the constraint: $\Sigma dxdirec \leq 4{,}5$ m

1000 runs were performed. The Pareto-Optimization (see also Fig. 11, which illustrates the principle of the Pareto-Optimization) automatically results in the maximum anchor length (5 x 5m) at the left upper corner and minimum anchor length (5 x 1m) at the right lower corner of the Pareto-front (see Fig. 12). The best design gives an accumulated anchor length of 11 m for 2.5 cm roof subsidence (see Fig. 13). Along the Pareto-Front all other optimum designs can be obtained. The minimum roof subsidence is reached with the anchor lengths *l* (1-5) = 5 m and the respective minima for the anchor distances. The interesting point is, that both, anchor length and location are strongly inhomogeneous for the optimum designs. This is caused by the inhomogeneous secondary stress field, the non-linearities of rock behavior (plastifications) and the interaction between rock mass and anchors. A more detailed analysis of the Pareto-optimized solutions is necessary in conjunction with practical restrictions (available anchor lengths, limitations in technology to install anchors of different lengths, incorporation of financial aspects in terms of anchors with different lengths etc.) to recommend an optimum anchor scheme. Finally, it would make sense to add a robustness analysis to investigate to what extent smaller unavoidable changes in the input parameters influence the output.

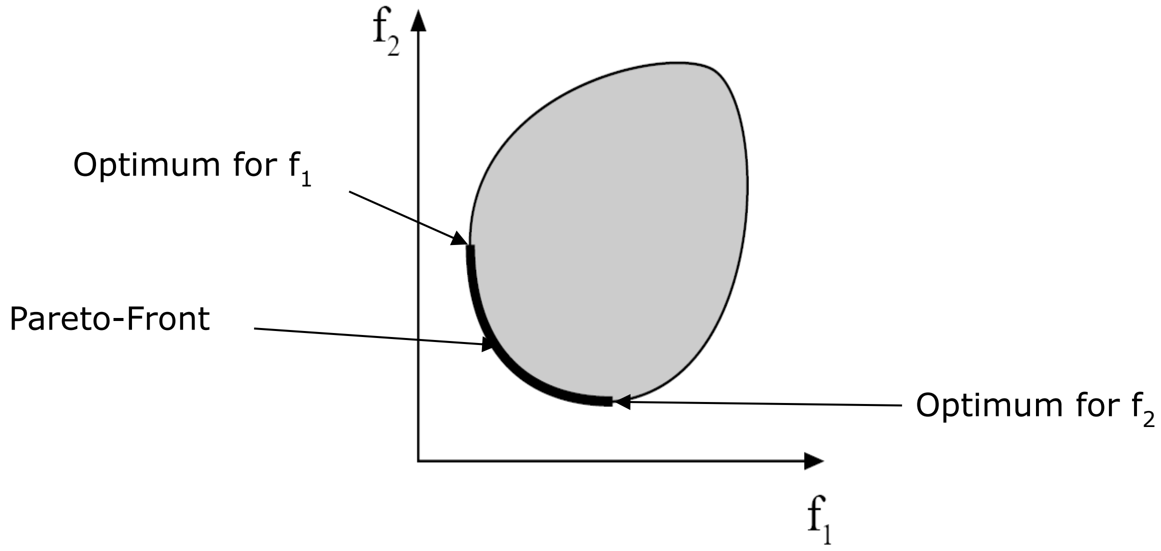

Fig. 11: Sketch to illustrate Pareto-Front and principle of Pareto-Optimization

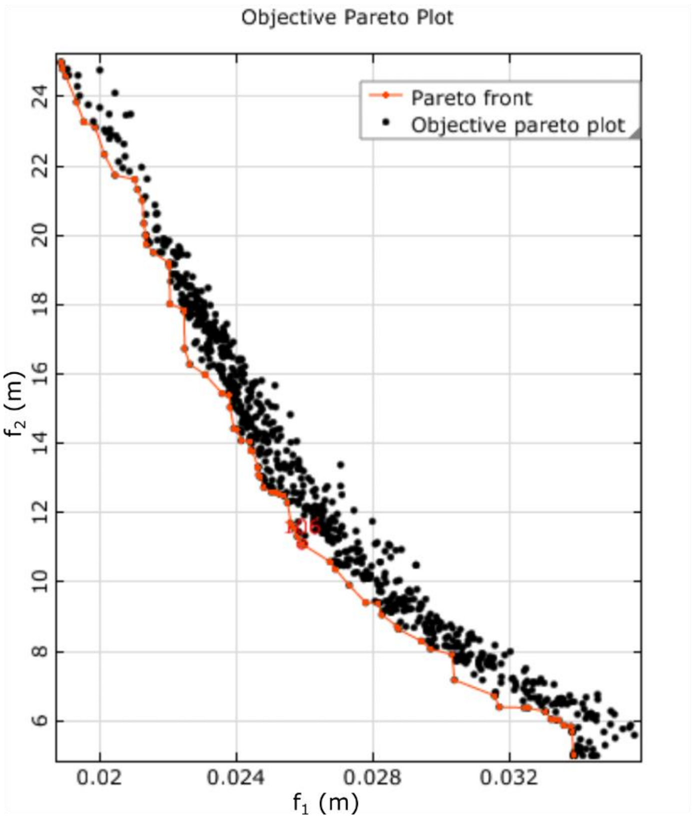

Fig. 12: Roof settlement f_1 versus accumulated anchor length f_2 with indication of best design (No. 106).

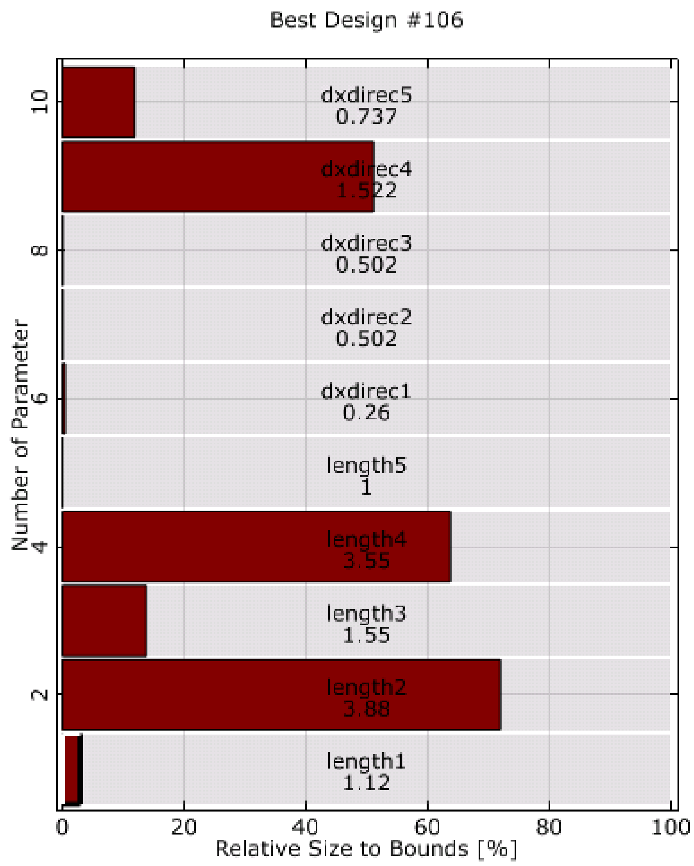

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Fig: 13: Parameters in terms of anchor length [m] and position [m] for best design (No. 106.)

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